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Dynamic Speckle - Interferometry of Micro-Displacements

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Abstract. The problem of the dynamics of speckles in the image plane of the object, caused by random movements of scattering centers is solved. We consider three cases: 1) during the observation the points move at random, but constant speeds, and 2) the relative displacement of any pair of points is a continuous random process, and 3) the motion of the centers is the sum of a deterministic movement and random displacement. For the cases 1) and 2) the characteristics of temporal and spectral autocorrelation function of the radiation intensity can be used for determining of individually and the average relative displacement of the centers, their dispersion and the relaxation time. For the case 3) is showed that under certain conditions, the optical signal contains a periodic component, the number of periods is proportional to the derivations of the deterministic displacements. The results of experiments conducted to test and application of theory are given.

Keywords: Micro-displacement, speckle, speckle dynamics, multi-beam interferometry, fatigue, ultrasound
PACS: 42.30.Ms

INTRODUCTION

At the present time to determine the macroscopic displacements well-known techniques such as holographic interferometry [1], correlation [2] and dynamic [3], speckle - interferometry is successfully applied. In [4] on a qualitative level, the reasons about the possibility of using the approaches taken in [1-3], to determine the microscopic displacements have expressed. Under the micro-displacements we mean random displacements, defined on the areas which size are comparable with the size of the structure of the material. Целью данной работы являлись теоретическое и экспериментальное обоснование метода определения микроскопических перемещений. The aim of this work is theoretical and experimental justification for the method of microscopic displacements determining.

THEORY

Object model and an expression for the radiation intensity

Let the source of coherent radiation with a wavelength λ , located at the point $\vec{s}$ illuminates the point scattering centers are located in the region S in the plane (*xoy*), as shown in Figure 1. Thin lens with focal length f and aperture diameter D, located in the plane ($\eta_x 0 \eta_z$), forms an image of the object in the plane ($q_x 0 q_y$). We consider a wave linearly polarized in one direction. We assume that the phase φ_j of the complex amplitude a_j of the wave scattered by the *j*-th center, is random, and at any point of the plane ($\eta_x 0 \eta_y$) waves arrive from all scattering centers. We obtain an expression for the intensity of radiation $I(\vec{q})$ at some point of the image plane of the object. For the complex amplitude $A(\vec{\eta})$ at any point $\vec{\eta}$ of the plane ($\eta_x 0 \eta_y$) we have:

$$A(\vec{\eta}) = \sum_{j=1}^{N} a_j \ , \tag{1}$$

where N - the number of scattering centers. The complex amplitude $A(\vec{q})$ of light at the point $\vec{q}$ we find, by summing the amplitudes of waves arriving from points in the plane ($\eta_x 0 \eta_y$) to the point $\vec{q}$, given the amplitude $P(\vec{\eta})$ and phase $\exp[i|\vec{\eta}|^2/(2f)]$ transmittance of the lens [5]:

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$$A(\vec{q}) = \int_{-\infty}^{+\infty}\!\!\int P(\vec{\eta})\, e^{\frac{i|\vec{\eta}|^2}{2f}}\, e^{ik\left|\vec{L}_q(\vec{\eta})\right|} \sum_{j=1}^{N} a_j\, d\eta_x d\eta_y\,, \qquad (2)$$

where i-imaginary unit, $\vec{L}_q(\vec{\eta})$- vector directed from point $\vec{\eta}$ to point $\vec{q}$. The relationship between the complex amplitude of light in the immediate vicinity of the point $\vec{r}_j$ and point $\vec{\eta}$ we take the same form as in [5]:

$$a_j(\vec{\eta}) = \sqrt{I_0(\vec{r}_j)}\, \xi(\vec{r}_j)\, e^{i\left\{k\left[\left|\vec{L}_s(\vec{r}_j+\vec{u}_j)\right| + \left|\vec{L}_\eta(\vec{r}_j+\vec{u}_j)\right|\right] + \varphi_j\right\}}, \qquad (3)$$

where $I_0 = I_0(\vec{r})$- the intensity distribution of the illuminating light, $\xi = \xi(\vec{r})$ - in general, the complex reflection coefficient, which takes into account the fraction of radiation emitted from point $\vec{r}$ to point $\vec{\eta}$, $\vec{L}_s(\vec{r})$- vector directed from point $\vec{r}$ to point $\vec{s}$, $\vec{L}_\eta(\vec{r})$- vector directed from point $\vec{r}$ to point $\vec{\eta}$, $\vec{u}_j$- a small displacement vector of the j-th center.

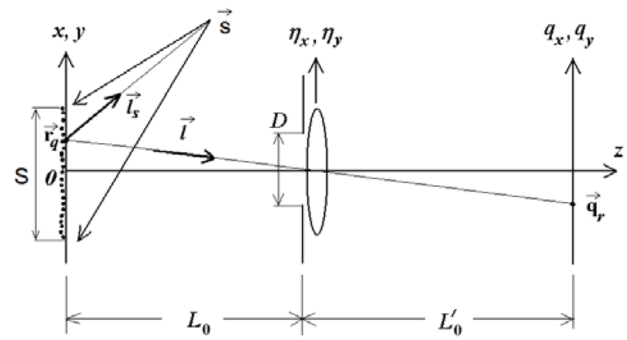


FIGURE 1. The optical system and the notation

Take an arbitrary point $\vec{q} = \vec{q}_r$ and the conjugate point $\vec{r}_q$. It is known that a wave coming from the point $\vec{r}_q$, as a result of light diffraction by the aperture D, forms an Airy's picture at the point $\vec{q}_r$. The diameter of the central spot b_s of picture is $1{,}22\lambda L'_0/D$, where L'_0- the distance from the lens to the plane $(q_x 0 q_y)$. Central spot corresponds to the area in the (xoy) plane. Diameter a_s of this area is b_s/m, where m is magnification of lens. It is known that the central spot of the Airy pattern accumulated for 85% of the wave energy transmitted through the lens. We shall neglect the energy that falls on areas outside of the diameter b_s. This, in turn, means that in point $\vec{q}_r$ there are waves coming only from the area a_s. Now let N - the number of centers in this area.

Further assume that the values of a_s, D and $|\vec{u}|$ are small compared with the distances from the object to the radiation source and the lens, as well as from the lens to the image plane. Considering the expression $\left|\vec{L}_s\right| = \left|\vec{L}_s(\vec{r}+\vec{u})\right|$ as a continuous function of $\vec{r}$ and $\vec{u}$, we expand it in Taylor series in a small neighborhood of $\vec{r} = \vec{r}_q$, $\vec{u}$=0, taking into account derivatives up to the first order. Similarly, we expand the expression $\left|\vec{L}_\eta(\vec{r}+\vec{u})\right|$ in the neighborhood of $\vec{r} = \vec{r}_q$, $\vec{\eta} = 0$ и $\vec{u}$=0, the expression $\left|\vec{L}_\eta(\vec{q})\right|$ in the neighborhood of $\vec{\eta} = 0$ и $\vec{q} = \vec{q}_r$. We have:

$$\left|\vec{L}_s(\vec{r}+\vec{u})\right| = \rho_s + l_{sx}\Delta x + l_{sy}\Delta y + l_{sx}u_x + l_{sy}u_y + l_{sz}u_z\,,$$

$$\left|\vec{L}_\eta(\vec{r}+\vec{u})\right| = \rho_1 + l_x\Delta x + l_y\Delta y + l_x u_x + l_y u_y + l_z u_z - l_x\eta_x - l_y\eta_y\,,$$

$$\left|\vec{L}_\eta(\vec{q})\right| = \rho_2 + l_x\eta_x + l_y\eta_y + l_x\Delta q_x + l_y\Delta q_y\,, \qquad (4)$$

where the following notation:

$\rho_1 = \sqrt{x_q^2 + y_q^2 + L_0^2}$, $\rho_2 = \sqrt{(q_{rx})^2 + (q_{ry})^2 + (L_0^/)^2}$, $\rho_s = \sqrt{\left(x_q - x_s\right)^2 + \left(y_q - y_s\right)^2 + z_s^2}$,

$\rho_1 = \sqrt{x_q^2 + y_q^2 + L_0^2}$, $\rho_2 = \sqrt{\left(q_{rx}\right)^2 + \left(q_{ry}\right)^2 + \left(L_0^/\right)^2}$, $\rho_s = \sqrt{\left(x_q - x_s\right)^2 + \left(y_q - y_s\right)^2 + \left(z_s\right)^2}$,

$\Delta x = x - x_q$, $\Delta y = y - y_q$, $\Delta q_x = q_x - q_{rx}$, $\Delta q_y = q_y - q_{ry}$,

$l_{sx} = x_s / \rho_s$, $l_{sy} = y_s / \rho_s$, $l_{sz} = z_s / \rho_s$,

$$l_x = x_q / \rho_1 = -q_{rx} / \rho_2 , l_y = y_q / \rho_1 = -q_{ry} / \rho_2 , l_z = -L_0 / \rho_1 . \qquad (5)$$

For simplicity, we assume that the area of diameter a_s uniformly illuminated, i.e. $I_0(\vec{r}) = const$ for all points of the field, and the value $\xi(\vec{r}) = 1$. Thus, by (3) - (5) we have:

$$A(\vec{q}) = \sqrt{I_0} \int_{-\infty}^{+\infty}\int P(\vec{\eta}) e^{i\frac{|\vec{\eta}|^2}{2f} + ik\left[\rho_s + \rho_1 + \rho_2 + \vec{l}\,\Delta\vec{q} - \vec{r}_q(\vec{l}_s + \vec{l})\right]} d\eta_x d\eta_y \sum_{j=1}^{N} e^{ik\left[\vec{u}_j(\vec{l}_s + \vec{l})\right]} e^{ik\vec{r}_j(\vec{l}_s + \vec{l}) + \varphi_j} =$$

$$= \sqrt{I_{01}} e^{i\psi_j} \sum_{j=1}^{N} e^{i\left\{k\left[\vec{u}_j(\vec{l}_s + \vec{l})\right] + \theta_j\right\}} \quad , \qquad (6)$$

where $\Delta\vec{q} = \Delta\vec{q}(\Delta q_x, \Delta q_y)$; $\vec{l}_s = \vec{l}_s(l_{sx}, l_{sy}, l_{sz})$ and $\vec{l} = \vec{l}(l_x, l_y, l_z)$ - the unit vectors directed from the point $\vec{r}_q$ to the radiation source and the observer, respectively, $\sqrt{I_{01}} e^{i\psi_j}$ the complex amplitude determines the expression on the left of the sum. For the radiation intensity at the point we have:

$$I(\vec{q}) = A(\vec{q}) A^*(\vec{q}) = I_{01} \sum_{j=1}^{N}\sum_{m=1}^{N} e^{i\left[k(\vec{u}_j - \vec{u}_m)(\vec{l}_s + \vec{l}) + \theta_j - \theta_m\right]} = I_{01} N + 2 I_{01} \sum_{\kappa=1}^{K} \cos\left[k\Delta\vec{u}_\kappa(\vec{l}_s + \vec{l}) + \Delta\theta_\kappa\right] =$$

$$= I_{01} N + 2 I_{01} \sum_{\kappa=1}^{K} \cos\left[\Delta\phi_\kappa + \Delta\theta_\kappa\right], \qquad (7)$$

$\Delta\vec{u}_\kappa$ - a vector of relative displacement of j-th pair of scattering centers, $\Delta\theta_\kappa = \theta_j - \theta_m$, $j \neq m$, $\kappa = 1, 2 \ldots K$, $K = N(N-1)/2$,

$$\Delta\phi_\kappa = k\Delta\vec{u}_\kappa\left(\vec{l}_s + \vec{l}\right) = 2k\Delta u_\kappa \cos(\alpha/2) = k\left[\Delta u_{x\kappa}(l_{sx} + l_x) + \Delta u_{y\kappa}(l_{sy} + l_y) + \Delta u_{z\kappa}(l_{sz} + l_z)\right], \qquad (8)$$

α - the angle between the vectors $\vec{l}_s$ and $\vec{l}$, Δu_κ - the projection of the vector Δu_κ on the bisector of the angle α , $\Delta u_{x\kappa}$, $\Delta u_{x\kappa}$, $\Delta u_{z\kappa}$ - components of the vector $\Delta\vec{u}_\kappa$.

Earlier [6], formula (7) for the case where the object of study was a thin transparent (phase) object was obtained. It was assumed that a thin object is located near the fixed scattering centers. For the case of the phase object $\Delta\phi_\kappa$ is equal to the phase difference of two waves passing through two different area of the object. In this paper, the value of $\Delta\phi_\kappa$ is given by (8). In [6] expressions for the temporal correlation and temporal spectral function of the random value of I at the point of observation was obtained. Our calculations showed that the temporal and spectral correlation function of the random variable I, defined by (7), up to notation coincide with the correlation and spectral functions found in [6]. Therefore, we use the results of [6], the detailed mathematical calculations can be found in this article.

Displacements of centers with constant velocities

As in [6], we consider two main cases. Let us first assume that the optical path difference $\Delta\phi_\kappa / k$ is greater than the wavelength λ, a velocity of change of $\Delta\phi_\kappa$ is random but constant during the observation period τ. In practice, this situation occurs when the observation time τ is small compared with the time correlation τ_0 of the random variable $\Delta\phi_\kappa$. We introduce the notation

$$\Delta\phi_\kappa = V_\kappa \tau,\; V_\kappa = k\Delta\vec{v}_\kappa\left(\vec{l}_s + \vec{l}\right) = 2k\Delta v_\kappa \cos(\alpha/2) = k\left[\Delta v_{x\kappa}(l_{sx} + l_x) + v_{y\kappa}(l_{sy} + l_y) + v_{z\kappa}(l_{sz} + l_z)\right], \quad (9)$$

where V_κ - the velocity of change of $\Delta\phi_\kappa$, $\Delta\vec{v}_\kappa$ - relative velocity vector to the κ-th pair of centers, $v_{x\kappa}, v_{y\kappa}, v_{z\kappa}$ - the components of the vector $\Delta\vec{v}_\kappa$, α - the angle between the vectors $\vec{l}_s$ and $\vec{l}$, Δv_κ - the projection of the vector $\Delta\vec{v}_\kappa$ on the bisector of the angle α.

According to [6], if at the time of observation τ is made sufficiently large number of measurements, the time spectrum of function $I = I(\tau)$, defined by (8) and (9), consists of narrow peaks. The height of the peaks is equal to I_0^2, the centers of the peaks occurs in the frequency $\omega_\kappa = V_\kappa$, and the number of peaks equals the number of K. From (9) it follows that at different angles of illumination or observation frequency will be different. Consequently, it is necessary to choose three different directions a, b, c of illumination or observation, at the same time register the three spectra. Then, for three values of frequency $\omega_\kappa^a, \omega_\kappa^b, \omega_\kappa^c$ and for formula (9), we can identify three components $v_{x\kappa}, v_{y\kappa}, v_{z\kappa}$ of the vector $\Delta\vec{v}_\kappa$. Then the formula $\Delta\vec{u}_\kappa = \Delta\vec{v}_\kappa \tau$ can determine the components of the vector $\Delta\vec{u}_\kappa$.

The relative displacement of two centers is a continuous random process

So, now let the process $\Delta u_\kappa = \Delta u_\kappa(t)$ and, consequently, the process $\Delta\phi_\kappa = \Delta\phi_\kappa(t)$ is a continuous random process. Again, we use the results of [6], where an expression for the time autocorrelation function of the radiation intensity I was obtained. We assume that the values $\Delta\theta_\kappa$ are distributed uniformly in the range π- to π+, for different values of κ the random variables $\Delta\phi_\kappa$ are statistically independent, and $\Delta\phi_\kappa = \Delta\phi_\kappa(\tau)$ in resolution area of the lens is uniform Gaussian random process. Then, for the time autocorrelation function of the random variable *I*, we have:

$$R_{1,2}(t_1,t_2) = I_{01}^2 N(N-1)\cos[\langle x_2\rangle - \langle x_1\rangle] \times e^{-\frac{1}{2}k_{11} - \frac{1}{2}k_{22} + k_{12}}, \quad (10)$$

where $\langle x_1\rangle$ and $\langle x_2\rangle$ - the average over the ensemble of objects values of $\Delta\phi_\kappa$ at the times t_1 and t_2, respectively, k_{11} and k_{22} - the dispersion of values $\Delta\phi_\kappa$ at the times t_1 and t_2, respectively, and k_{12} - the correlation coefficient of the $\Delta\phi_\kappa$ at the times t_1 and t_2. In general, the expression $R_{1,2}(t_1,t_2)$ depends on the choice of origin and, therefore, the process $I = I(t)$ is not stationary. The expansion of this function to the periodic components can be made, for example, involving the wavelet - transformation. The following are the special cases for which we can apply the usual Fourier spectral analysis of signals. Note also that the phase difference and the relative displacement of the centers are connected by the formula (8).

The mean values of the phase differences are zero

Consider the important case in practice, when $\langle x_1\rangle = \langle x_2\rangle$, or the average values of relative displacement is zero. We also assume that the process $\Delta\phi_\kappa = \Delta\phi_\kappa(\tau)$ is stationary. Then for the normalized autocorrelation function we have:

$$\eta(\tau) = e^{-k_{11} + k_{11}\rho_{12}(\tau)}, \quad (11)$$

where $\rho_{12}(\tau)$- the normalized autocorrelation function of the relative displacements. Consider the features of the expression (11) at $k_{11} << 1$, i.e. when the standard deviations of the relative displacements are small compared to the wavelength of radiation. Then the value of η in (11) is close to unity. Suppose τ_0 is time correlation of the value Δu_κ. When at $\tau >> \tau_0$ the value η drops rapidly to a level close to $\eta^* = \exp(-k_{11})$. For example, if $\rho_{12}(\tau)$- it is the Gaussian function, then at τ_0 = 5 s, even at $\tau \approx 2{,}5$ τ_0 value of η is different from the value η^* less than 1%. If it is easy to show that

$$\eta'(\tau) = \frac{\eta(\tau) - \eta^*}{1 - \eta^*} \approx \rho_{12}(\tau). \tag{12}$$

Thus, from the experimental dependence $\eta = \eta(\tau)$ easily can be define a function $\rho_{12}(\tau)$, hence, the relaxation time τ_0 of relative micro-displacements. According to the plateau value η^* of the function $\eta = \eta(\tau)$ can determine the variance or standard deviation of the relative micro- displacements.

Now consider the case when $k_{11} \geq 1$. In particular, this version takes place when the standard deviation of displacements is greater then λ. The expression for the time of the spectral function has so far been obtained for the case when the function is a Gaussian function. In [6] showed that in this case, the function describing the dynamics of the fluctuating component of the speckles is a function of Gauss. Then, the normalized spectral function of the intensity fluctuations is a function of Gauss. Consequently, one can easily obtain the relation between the correlation time of the radiation intensity τ_κ and correlation time of relative displacement τ_0, as well as between the width of the temporal spectrum of the relative displacement ω_0 and the width of the temporal spectrum ω_κ of the radiation intensity. Detailed calculations of these ratios are given in [6]. We have:

$$\tau_k = \tau_0 \left[\ln \frac{k_{11}}{\ln\left\{\left[1 - e^{-k_{11}} + e^{-k_{11}+1}\right] - 1\right\}} \right]^{0,5}, \tag{12}$$

$$\omega_k = \omega_0 \left[\ln \frac{k_{11}}{\ln\left\{\left[1 - e^{-k_{11}} + e^{-k_{11}+1}\right] - 1\right\}} \right]^{-0,5}. \tag{13}$$

Figure 2 the combined dependencies of τ_κ / τ_0 and ω_κ / ω_0 on k_{11} shows. It follows from them that the time spectra brightness of the speckle and time spectra of relative displacements are the same, while at $k_{11} \geq 2$ there is almost a linear relationship between ω_κ and ω_0, and their ratio is k_{11}.

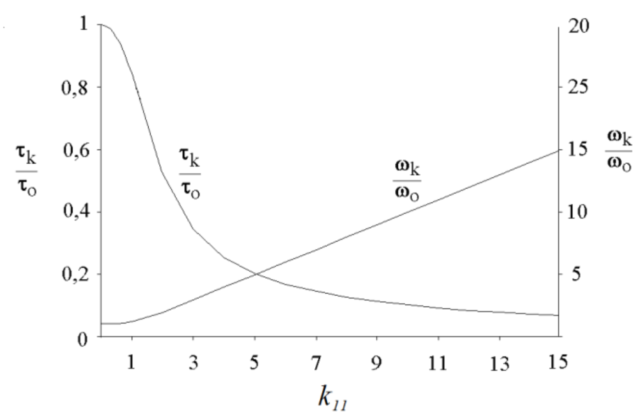


FIGURE 2. The combined dependences of $\tau_k/\tau_0 = \tau_k/\tau_0(k_{11})$, $\omega_k/\omega_0 = \omega_k/\omega_0(k_{11})$

THE EFFECT OF PERIODIC CHANGE IN THE BRIGHTNESS OF SPECKLES

We again consider the equation (4) and (5). Assume further that the displacements of points of the medium are the sum of the two types of movement - the deterministic $\vec{u}^d$ and random $\vec{u}^r$ displacement. We believe that the displacement $\vec{u}^d$ caused by the macroscopic processes in a_s, but the random movement - due to of microscopic processes. Assuming that second derivatives of the deterministic displacements are zero, we expand them in Taylor series in the vicinity $\vec{r} = \vec{r}_q$. Omitting the index d, we have:

$$u_x = u_{0x} + \frac{\partial u_x}{\partial x}(x - x_q) + \frac{\partial u_x}{\partial y}(y - y_q) = u_{0x} + \frac{\partial u_x}{\partial x}x + \frac{\partial u_x}{\partial y}y - \frac{\partial u_x}{\partial x}x_q - \frac{\partial u_x}{\partial y}y_q = u_{0x} + A_x - B_x\,, \quad (14)$$

where u_{0x} - the projection of the translational movement $\vec{u}_0$ of a point $\vec{r} = \vec{r}_q$ on the x-axis,

$$A_x = grad\, u_x \cdot \vec{r}\,,\ B_x = grad\, u_x \cdot \vec{r}_q\ , \quad (15)$$

Similarly, we have:

$u_y = u_{0y} + A_y - B_y$, $u_z = u_{0z} + A_z - B_z$, where

$$A_y = (grad\, u_y)\cdot \vec{r}\,,\ B_y = grad\, u_y \cdot \vec{r}_q\ ,\ A_z = grad\, u_z \cdot \vec{r}\,,\ B_z = grad\, u_z \cdot \vec{r}_q\ . \quad (16)$$

Again, for simplicity, we assume that the object is illuminated uniformly, i.e. $I_0(\vec{r}) = const$ for all points of the object, and the value $\xi(\vec{r}) = 1$. Thus, taking into account (4) - (5) and (14) - (16) instead of (3) we have:

$$A(\vec{q}) = \sqrt{I_0}\int\!\!\int_{-\infty}^{+\infty} P(\vec{\eta}) e^{i\frac{|\vec{\eta}|^2}{2f} + ik\left[\rho_s + \rho_1 + \rho_2 + \vec{l}\Delta\vec{q} - \vec{r}_q(\vec{l}_s + \vec{l}) + \vec{u}_0(\vec{l}_s + \vec{l}) - \vec{B}(\vec{l}_s + \vec{l})\right]} \cdot$$

$$\cdot \sum_{j=1}^{N} e^{ik\left[\vec{A}_j(\vec{l}_s + \vec{l})\right]} e^{ik\vec{r}_j(\vec{l}_s + \vec{l}) + \varphi_j + u_j^r(\vec{l}_s + \vec{l})} d\eta_x d\eta_y =$$

$$= \sqrt{I_{01}} e^{i\psi} \sum_{j=1}^{N} e^{ik\left[\vec{A}_j(\vec{l}_s + \vec{l})\right]} e^{ik\vec{r}_j(\vec{l}_s + \vec{l}) + u_j^r(\vec{l}_s + \vec{l}) + \varphi_j} = \sqrt{I_{01}} e^{i\psi} \sum_{j=1}^{N} e^{ik\left[\vec{A}_j(\vec{l}_s + \vec{l})\right]} e^{i\theta_j}\ , \quad (17)$$

where $\vec{l}_s = \vec{l}_s(l_{sx}, l_{sy}, l_{sz})$, $\vec{l} = \vec{l}(l_x, l_y, l_z)$ - again the unit vectors directed from the point $\vec{r}_q$ to the radiation source and to the observer, respectively, the complex amplitude $\sqrt{I_{01}} e^{i\psi}$ determines of independent of the index j expression before the summation, the value $\theta_j = k(\vec{r}_j(\vec{l}_s + \vec{l}) + \vec{u}_j^r(\vec{l}_s + \vec{l}) + \varphi_j$ united the random coordinates, the random displacement and random initial phase of wave of the j - th center. Suppose now that there are K scattering centers, the coordinates x_j are located at equal distances, i.e. $x_j = j \cdot \Delta x$, the coordinates y_j are a random values. In this case, for the right-hand side of (17) we have:

$$\sqrt{I_{01}} e^{i\psi} \{ \sum_{j=1}^{K} e^{ik\left[\left(\frac{\partial u_x}{\partial x}(l_{sx} + l_x) + \frac{\partial u_y}{\partial x}(l_{sy} + l_y) + \frac{\partial u_z}{\partial x}(l_{sz} + l_z)\right)\Delta x \cdot j\right]} e^{i\theta_j} + \sum_{m=1}^{N-K} e^{ik\left[\vec{A}_m(\vec{l}_s + \vec{l})\right]} e^{i\theta_m} \}\ , \quad (18)$$

where the value θ_j is now also contains a random coordinate y_j included in the expression of vector $\vec{A}_j$. Suppose now that we have chosen such K scattering centers, for which the values θ_j are the same and equals θ_0. Instead of (18) we have:

$$\sqrt{I_{01}} e^{i\Psi} \{ e^{ik\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l})\right)\Delta x \cdot\right]} \cdot \frac{1 - e^{ik\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l})\right)\Delta x \cdot K\right]}}{1 - e^{ik\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l})\right)\Delta x \cdot\right]}} e^{i\Theta_0} + \sum_{m=1}^{N-K} e^{ik\left[\vec{A}_m(\vec{l}_s + \vec{l})\right]} e^{i\Theta_m} \} =$$

$$= \sqrt{I_{01}} e^{i\psi} \cdot \left\{ \frac{e^{i\frac{k}{2}\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l}\,)\right)\Delta x \cdot (K+1)\right]} \cdot \sin\left[\frac{k}{2}\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l}\,)\right)\Delta x \cdot K\right]\right]}{\cdot \sin\left[\frac{k}{2}\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l}\,)\right)\Delta x\right]\right]} + \sum_{m=1}^{N-K} e^{ik\left[\vec{A}_m(\vec{l}_s + \vec{l}\,)\right]} e^{i\theta_m} \right\}. \quad (19)$$

For the radiation intensity at the point $\vec{q}$ we have:

$$I(\vec{q}) = A(\vec{q})A^*(\vec{q}) = I_{01} \frac{\sin^2\left[\frac{k}{2}\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l}\,)\right)\Delta x \cdot K\right]\right]}{\sin^2\left[\frac{k}{2}\left[\left(\frac{\partial \vec{u}}{\partial x}(\vec{l}_s + \vec{l}\,)\right)\Delta x\right]\right]} + I_2 . \quad (20)$$

where - I_2 the sum of values that contain random displacement. Thus, the intensity of radiation at a point $\vec{q}$ can be represented as the sum of two terms. The first term is a periodic function, the argument of which depends only on the derivatives of the deterministic displacements along the coordinate x. The second term depends on the random motion of scattering centers.

Conducted by now the analysis leads to several conclusions. If there are scattering centers, the coordinates y_j are located at equal distances, and the corresponding phases are identical, then the expression (20) will be one more term, which depends on the derivatives of the displacements along the coordinate y. If the size of the illuminated object is such that we can neglect the second derivatives of the displacements of the coordinates, then (17) - (20) are also valid for the free space geometry.

EXPERIMENTS

To check the correctness of the formulas (11) and (12) we have made the experiment, in which a random displacements of scattering centers were created by means of high-cycle fatigue of metal. A flat sample of carbon steel 50 periodically was load by cantilever bending. Loading frequency was equal to 50 Hz, number of cycles- up to 1 million 200 thousand, the amplitude of the cycle has changed from 0.2 up to 0.82 $\sigma_{0,2}$, where $\sigma_{0,2}$ is the yield strength of the steel 50. Area near a dangerous section illuminated with a laser with the wave length of λ = 0,655μm and a power of 20 mW. Images with magnification m = 0,1 captured at a certain phase fluctuations of an object, injected into the computer at frequency of about 10 Hz. The minimum size of the speckle in the plane of the object image was equal to 40 microns. Movies for about 20 - 60 seconds recorded at different stages of the test sample. To determine the value of η took the brightness values of the speckle at one observation point (pixels), but in different moments of time t_1 and $t_1 + \tau$. Brightness value $I_1(t_1)$ corresponded to the beginning of the film, $I_2(t_1 + \tau)$ - to a frame in time $t_1 + \tau$. As one of the "object "of the ensemble of objects considered part of the surface size $a_s = b_s / m$, and as an ensemble of objects considered the totality of the areas of size a_s located in the region of the surface of 1×4 mm. The value of η was determined by the formula:

$$\eta(\tau) = \frac{\left\langle \left[I_1(t_1) - \langle I_1(t_1)\rangle\right]\left[I_2 - \langle I_2(t_1+\tau)\rangle\right]\right\rangle}{\sigma_1 \times \sigma_2}, \quad (21)$$

where the angular brackets mean the arithmetic average of the pointed ensemble of objects, σ_1 and σ_2 - mean-square deviation values of $I_1(t_1)$ and $I_2(t_1 + \tau)$.

In Figure 3a and 3b the typical experimental dependence, corresponding to the various amplitudes of the cycle is shown. For comparison on Figure 3c dependence for the phase object taken from work [6] is given. In the work [6] the speckle dynamics was created by the movement of the glass plate, roughness it was known. Point - experiment, dashed line - theory. The latter was carried out by the method of least squares by substituting the values of τ_0 and k_{11}. It was assumed that the function $\rho_{12}(\tau)$ is a function of Gauss. In Figure 3a deviation theory of experiment is 5%, Figure3b - 15%. Theoretical and experimental data, presented at the Figure 3c differed by 2.5%. Value of

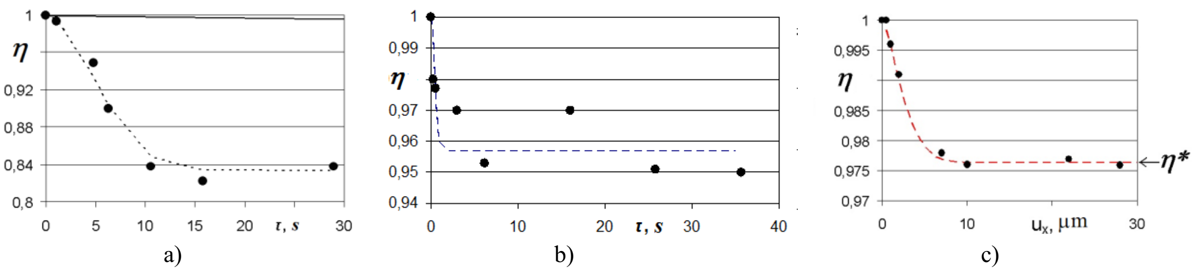


FIGURE 3. Dependence of η on τ (u_x). a,b - metal fatigue, c-displacement of the glass plate.

standard deviation of the optical path difference, found by the formula (11) and of the roughness, differed in [6] by 5%. This error can currently take for a minimum accuracy of the determination of the value $\sqrt{k_{11}}$ by formula (11).

Analysis of values τ_0 and $s_z = \lambda\sqrt{k_{11}}/(4\pi)$, obtained when tested for fatigue, has shown the following. Value τ_0, which characterizes the rate of change of the relative micro-displacements, when you change the amplitude of the cycle from 0,2 up to 0.82 $\sigma_{0,2}$, reduced by the logarithmic law from 60 up to 0.5 sec. In two cases at the beginning of the test and in one case before the destruction of the sample found the emissions of a value η^* equal to 0.8, the 0,91 and of 0.95, respectively. In other cases, when you change the cycle amplitude within the limits from 0,2 up to 0.82 $\sigma_{0,2}$ and the number of loading cycles N up to 1 million 200 thousand, the value η^* close to the average value equal to the 0,992 ± 0,003. Value η^* = 0,992 corresponds to the standard deviation of relative displacement, equal to 5 ± 1 nm. At the present time, in contrast to the work [6], the value of s_z using an alternative method is not defined. However, found that in areas with minimal initial roughness of Ra = 4-20 nm average roughness increases linearly with increase of N, the coefficient of correlation between the Ra and N is equal to 0,96.

Figure 4 gives a space - time picture, showing the effect of a periodic change the brightness of speckle. The

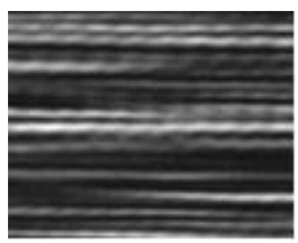
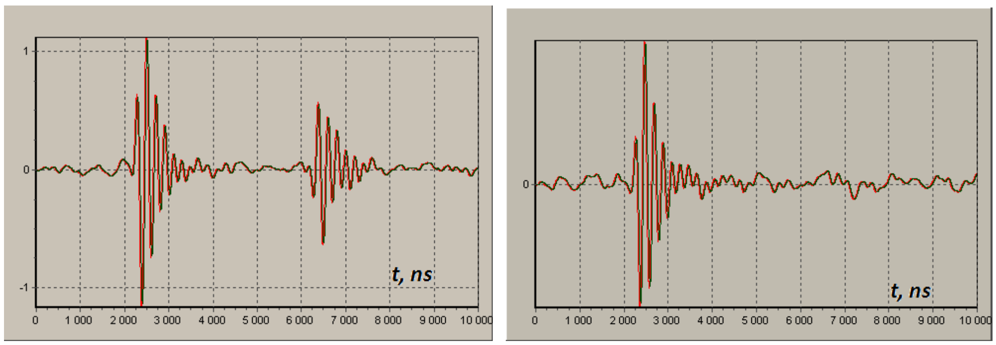


FIGURE 4. Space - time picture

picture was detected in the image plane of the working part of the sample of steel 03Г21Х13. Sample loaded with tension up to destruction at a speed of approximately 3 μm/s. The vertical line shows the speckle brightness along a line (about 2 mm on specimen) drawn parallel to the axis of the specimen, the horizontal is situated time (about 17 second). A small portion was taken to illustrate the effect. Effect was observed from the beginning of plastic deformation before the destruction, including in the plastic strain localization. On Figure 4 observation and lighting were close to the normal of the surface. According to formula (20), the number of periods in proportion to the turn of the surface around the lines, perpendicular to the axis of the specimen. Observation shows that if in the sample there has been a significant change of its form, it is observed the beating of different periodicities. This is also consistent with the theory.

Let macroscopic displacements of the points of the surface occur in (xoz) plane, displacements are small compared with the wavelength λ, random microscopic displacements equal to zero. Let the surface normal coincides with the axis oz. Such a situation takes place in the propagation of elastic Rayleigh wave. Then in the expression (20) speckle brightness will be proportional to the derivative $\partial u_z / \partial x$. Figure 5a shows the dependences of the shift of two parts of the surface from time during the passage through them of the Rayleigh wave. Waveform was fixed according to the methods described in the works [7,8]. Two areas of diameter 40 μm, located at a distance of about 10 mm, illuminated by the normal. Speckle waves reflected in the normal direction, combined. In the area of overlap of the two speckle was a photodiode. According to [7], in Figure 5a of the optical signal is formed as a result of interference of two waves and is proportional to the relative displacement Δu_z of the two areas. With a displacement of one part of the second section was motionless. Figure 5b shows the oscillogram obtained at the closing of a laser beam. The laser beam, far from the generator, was closed. We found that if we use two beams, but the effect of multipath interference is absent, then the signals to Figure 5a and 5b are shifted in phase exactly on 90°, which is consistent with theory. The physical essence of the effect shown in Figure 5b, is simple.



a) b)

FIGURE 5. Oscillograms corresponding to the Rayleigh wave: a) two-beam method, b) single-beam method.

Let scattering centers located at equal distances along the axis ox. Let centers are located either on the crest or on the trough of the Rayleigh wave. Let the initial phase of the corresponding waves are equal. Then the relative changes of phases of the reflected waves will be equal to zero, the brightness at the point of observation will be unchanged. If illuminated area will be between the crest and the trough of the wave, then there is a situation, as in the diffraction grating. As a result of interference of many waves the speckle brightness will be changed proportionally $\partial u_z / \partial x$. Figure 6 shows the dependence of the displacement of the area, illuminated by a laser, from

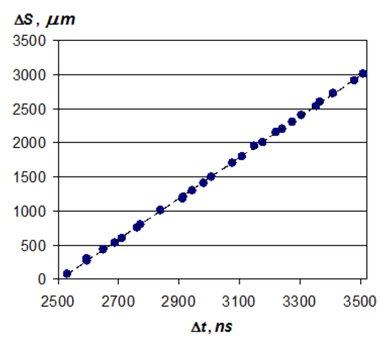


FIGURE 6. The relationship between the position of the illuminated area and the propagation time of the Rayleigh wave

the time coordinate of the signal peak. The data obtained with the help of a one laser beam. The slope of the dependence is equal to the velocity of the Rayleigh wave, equal to 2994 ± 10 m/s. The advantage of this velocity measurement method is the possibility of determining the velocity on a region size of 1 mm or less. Error of definition of velocity compared with two - beam methodology is reduced by about a five times.

DISCUSSION

In this article theoretically connection between random displacements of scattering centers, located on the surface of the object and the dynamics of speckle in the plane of its image has been studied. A detailed version of the small relative movements and application of the obtained results to the study of multi-cycle fatigue considered. According to the literary data, found the movement of scattering centers and an increase in roughness are reflections of the process of generation of dislocations in metals with fatigue. The relative gradual reduction in the value η up to the value η^* of the plateau corresponds to the process of annihilation of dislocations. The increase in roughness is due to the dynamics of dislocations in the metal. Their dynamics leads to the appearance of various dissipative structures, changing roughness. The methodology of micro-displacements may be useful for the creation of physical models and theories of fracture of fatigue, meeting the requirements of engineering practice. It is important that there is a possibility to define the relative movements not only normal to the surface, but also in the plane of the object.

Multi-beam dynamic interference of waves, leading to periodic change the brightness of the speckle, can be used as an alternative to the methods of photo elasticity and photo plasticity. Advantage of this method is the possibility to determine the derivatives of the displacements. The method can be used for registration of ultrasonic oscillations.

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